MAY 2020

Roll No.

Total No. of Questions : 08

Total No. of Pages : 02

B.Tech. (CE/ME/ECE/EE) (2018 & Onward) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Time : 2 Hrs.

Max. Marks : 30

INSTRUCTIONS TO CANDIDATES :

1. Attempt any FIVE question(s), each question carries 6 marks.

1. a) Expand $f(x) = e^{a\sin^{-1}x}$ in ascending powers of x upto x^4.

   b) Evaluate $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$.

2. a) Find the maximum value of $\sin^p x \cos^q x$.

   b) Find the volume of the solid generated by revolving the curve $xy^2 = 4(2-x)$ about y-axis.

3. a) If $u(x,y) = \frac{x^2+y^2}{x+y}$, then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$.

   b) Find the maximum and minimum values of $x^3 + 3xy^2 - 3y^2 + 4$.

4. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dxdy$ after changing into polar coordinates.

   b) Evaluate $\iint_R (x+y)\,dxdy$ where R is the region bounded by $x=0, x=2, y=x, y=2+x$.

5. a) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ when $|p| \le 1$.

   b) Examine the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots$ for convergence.

(S1)-44

10. Find the determinant of the following matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & 7 \end{bmatrix}$

SECTION-B

11. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$. Show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

12. Evaluate $\iint \frac{xy\,dx\,dy}{(1-y^2)^{1/2}}$ over the first quadrant of the circle $x^2 + y^2 = 1$.

13. Test the convergence of the series $\sum \frac{4.7......(3n+1)x^n}{n!}$.

14. Verify if the matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal and hence find its inverse.

SECTION-C

15. Find the maximum and minimum value of $x^3 + y^3 - 3axy$.

16. a) Solve the simultaneous equations $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$.

b) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$.

17 a) Find the area of the surface of revolution generated by revolving the curve $x = y^3$ from $y = 0$ to $y = 2$.

b) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$.

18. a) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^2}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + ...$

b) Find the Maclaurin's series of $f(x) = \cos x$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

Subject - MATHEMATICS-I (BTAM-101-18)
PTU. May 2020 (OBE)

Q1 (a) Expand $f(x) = e^{a\sin^{-1}x}$ in ascending powers of x upto x^4.

$$f'(x) = e^{a\sin^{-1}x} \cdot \frac{a}{\sqrt{1-x^2}} = a e^{a\sin^{-1}x} (1-x^2)^{-1/2}$$

$$f''(x) = a\left[e^{a\sin^{-1}x} \cdot \frac{a}{\sqrt{(1-x^2)}} (1-x^2)^{-1/2} + e^{a\sin^{-1}x} \left(-\frac{1}{2}(1-x^2)^{-3/2}(-2x)\right)\right]$$

$$f''(x) = e^{a\sin^{-1}x}\left[\frac{a^2}{(1-x^2)} + ax(1-x^2)^{-3/2}\right]$$

$$f'''(x) = e^{a\sin^{-1}x} \frac{a}{\sqrt{1-x^2}}\left[\frac{a^2}{(1-x^2)} + \frac{ax}{(1-x^2)^{3/2}}\right]$$

$$+ e^{a\sin^{-1}x}\left[\frac{-a^2}{(1-x^2)^2}(-2x) + \frac{a(1-x^2)^{3/2} - x\frac{3}{2}(1-x^2)^{1/2}(-2x)}{(1-x^2)^3}\right]$$

Now $f(0) = 1$

$f'(0) = a$, $f''(0) = a^2$, $f'''(0) = a^3$, $f^{iv}(0) = a^4$

By Maclaurin's theorem

$$f(x) = e^{a\sin^{-1}x} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0)$$

$$e^{a\sin^{-1}x} = 1 + ax + \frac{a^2x^2}{2} + \frac{x^3}{6}a^3 + \frac{a^4x^4}{24} + \cdots$$

Q1 (b) Evaluate $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

Sol^n:

$$= \lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$$

$$= \lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x^2} \left(\frac{x}{\sin x}\right) \qquad 1 \because \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x^2} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{e^x + e^{-x} - \frac{2}{1+x}}{2x}$$

$$= \lim_{x \to 0} \frac{e^x + e^{-x} + \frac{2}{(1+x)^2}}{2}$$

$$= \frac{e^0 - e^0 + \frac{2}{1}}{2} = 2/2 = 1$$

Value Theore

Q 2(b) Find the volume of solid generated by revolving the curve $xy^2 = 4(2-x)$ about y-axis.

Sol^n: Given curve is

$$xy^2 = 4(2-x) \Rightarrow y^2 = \frac{4(2-x)}{x}$$

$$y^2 = \frac{8}{x} - 4$$

About y-axis, $x = 0$,

at $x=0$, $y \to \infty$ $\quad \left| \because y^2 = \frac{8}{0} - 4,\ y \to \infty \right|$

$\therefore -\infty \le y \le \infty$

Now Volume about y-axis

$$\text{Volume} = \int_{-\infty}^{\infty} \pi x^2 \, dy$$

$$\left| \begin{array}{l} xy^2 - 8 + 4x = 0 \\ x(y^2+4) = 8 \\ x = \frac{8}{(y^2+4)} \end{array} \right.$$

$$= \pi \int_{-\infty}^{\infty} \frac{8^2}{(y^2+4)^2} dy$$

$\because$ f(y) is even

$$= 64\pi \cdot 2 \int_0^{\infty} \frac{1}{(y^2+4)^2} dy$$

Put $y = 2\tan\theta$ $\quad$ at $y = 0$, $\theta = 0$

$dy = 2\sec^2\theta \, d\theta$ $\quad$ $y = \infty$, $\theta = \pi/2$

$$\text{Volume} = 128\pi \int_0^{\pi/2} \frac{2\sec^2\theta \, d\theta}{16(\tan^2\theta+1)^2}$$

$$= \frac{128\pi \times 2}{16} \int_0^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^4\theta}$$

$$= \frac{128\pi}{8} \int_0^{\pi/2} \cos^2\theta \, d\theta .$$

$$V = \frac{128}{8}\pi \cdot \frac{\pi}{2} \cdot \frac{1}{2} = \underline{4\pi^2}$$

Sec(b)

pap

Q2 (a) Find the maximum value of $\sin^p x \cos^q x$

Sol^n: Let $f(x) = \sin^p x \cos^q x$

$$f'(x) = \sin^p x \, q \cos^{q-1} x \, (-\sin x) + p \sin^{p-1} x \cos x \cos^q x$$

For stationary point

$$f'(x) = 0$$

$$-q \sin^p x \cos^{q-1} x \sin x + p \sin^{p-1} x \cos x \cos^q x = 0$$

$$q \sin^{p+1} x \cos^{q-1} x = p \sin^{p-1} x \cos^{q+1} x$$

$$\sin^2 x = \frac{p}{q} \cos^2 x$$

$$\tan^2 x = p/q$$

$$\tan x = \pm\sqrt{p/q}$$

Q3

Sol

Q3 a) If $u(x,y) = \frac{x^2+y^2}{x+y}$

then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$

Solⁿ: Given $u(x,y) = \frac{x^2+y^2}{x+y}$

$$\frac{\partial u}{\partial x} = \frac{(x+y)2x - (x^2+y^2)}{(x+y)^2} = \frac{2x^2+2xy-x^2-y^2}{(x+y)^2}$$

$$\frac{\partial u}{\partial x} = \frac{x^2-y^2+2xy}{(x+y)^2}$$

Now $\frac{\partial u}{\partial y} = \frac{(x+y)2y - (x^2+y^2)}{(x+y)^2} = \frac{2xy+2y^2-x^2-y^2}{(x+y)^2}$

$$\frac{\partial u}{\partial y} = \frac{2xy+y^2-x^2}{(x+y)^2}$$

Now $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = \left(\frac{x^2-y^2+2xy-(2xy+y^2-x^2)}{(x+y)^2}\right)^2$

$$= \left(\frac{2(x^2-y^2)}{(x+y)^2}\right)^2$$

$$= \frac{4(x^2-y^2)^2}{(x+y)^2(x+y)^2} = 4\frac{\cancel{(x+y)^2}(x-y)^2}{\cancel{(x+y)^2}(x+y)^2} = \frac{4(x-y)^2}{(x+y)^2}$$

Also $4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right) = 4\left(1 - \frac{(x^2-y^2+2xy)}{(x+y)^2} - \frac{(2xy+y^2-x^2)}{(x+y)^2}\right)$

$$= 4\left(\frac{\cancel{x^2}+\cancel{y^2}+\cancel{2xy}-\cancel{x^2}+y^2-\cancel{2xy}-2xy-\cancel{y^2}+x^2}{(x+y)^2}\right)$$

$$= \frac{4}{(x+y)^2}(x-y)^2$$

$\therefore \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$ hence proved

3(b) Find the maximum & minimum values of $x^3+3yx^2-3y^2+4$

Sol^n: Let $f(x,y) = x^3+3xy^2-3y^2+4$

For stationary pts

$\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2+3y^2=0 \Rightarrow x=-y$

$\frac{\partial f}{\partial y} = 6xy-6y=0 \Rightarrow 6y(x-1)=0$

or $y=0$ or $x=1$

At $y=0$, $x=0$, $(0,0)$

At $x=1$, $y=-1$, $(1,-1)$

Now $r = \frac{\partial^2 f}{\partial x^2} = 6x$

$s = \frac{\partial^2 f}{\partial x \partial y} = 6y$

$t = \frac{\partial^2 f}{\partial y^2} = 6(x-1)$

At $(0,0)$, $rt-s^2 = 0-0=0$

Further investigation is needed

$\Delta f = f(0+h, 0+k) - f(0,0)$

$= h^3+3hk^2-3k^2+\cancel{4}-\cancel{4}$

$= h^3+3hk^2-3k^2$

$\Delta f = -3k^2$

$\because$ h & k are very small neglecting higher terms

for $k>0$ or $k<0$

$\Delta f < 0$

$\therefore$ $f(x,y)$ has local maxima at $(0,0)$

Maximum Value of $f(0,0) = 4$

At $(1,-1)$

$r = 6x = 6$

$s = 6y = -6$

$t = 6(x-1) = 0$

$\therefore rt - s^2 = 0 - (-6)^2$

$rt - s^2 = -36$

$rt - s^2 < 0$

$\therefore (1,-1)$ is a saddle point

Q4: a) Evaluate

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dx\,dy$$

after changing into polar co-ordinates.

Soln:

Let $I = \int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dx\,dy$

in polar co-ordinate system

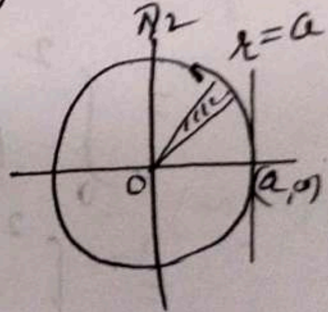
$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2(\cos^2\theta + \sin^2\theta)$

$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$



Here $0 \le r \le a$

$0 \le \theta \le \pi/2$

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\,dx\,dy = \int_0^{\pi/2}\int_0^a r^2 \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^a d\theta$$

$$= \int_0^{\pi/2} \frac{a^4}{4}\,d\theta$$

$$= \frac{a^4}{4}\left[\theta\right]_0^{\pi/2} = \frac{a^4}{4}\cdot\frac{\pi}{2}$$

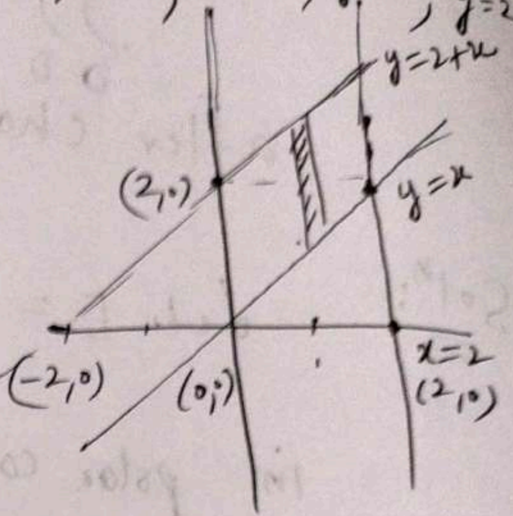
$$I = \frac{\pi a^4}{8}$$

Q4 (b) Evaluate $\iint_R (x+y)\, dx\, dy$

R is region bdd by $x=0, x=2, y=x, y=2$

$0 \le x \le 2$

$x \le y \le 2+x$



$$\int_0^2 \int_x^{2+x} (x+y)\, dy\, dx$$

$$= \int_0^2 \left[xy + \frac{y^2}{2} \right]_x^{2+x} dx$$

$$= \int_0^2 \left[x(2+x) + \frac{(2+x)^2}{2} - x^2 - \frac{x^2}{2} \right] dx$$

$$= \int_0^2 \left(2x + x^2 + \frac{4+x^2+4x}{2} - \frac{3x^2}{2} \right) dx$$

$$= \int_0^2 (2+4x)\, dx$$

$$= \left[2x + \frac{4x^2}{2} \right]_0^2$$

$$= [2+8]$$

$$\boxed{I = 16}$$

Q5 (ii) Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \text{When } p \le 1$$

Solution: Here $u_n = \frac{1}{n^p}$

Let $f(x) = \frac{1}{x^p}$

For $x \geq 1$, $f(x)$ is +ve & monotonically decreasing

$\therefore$ Cauchy's Integral test is applicable

- For $p \neq 1$

$$\int_1^\infty f(x)\,dx = \int_1^\infty \frac{1}{x^p}\,dx = \int_1^\infty x^{-p}\,dx = \left[\frac{x^{-p+1}}{-p+1}\right]_1^\infty$$

(i) When $p<1$, $1-p$ is +ve

$$\therefore \int_1^\infty \frac{1}{x^p}\,dx = \frac{1}{1-p}\left[x^{1-p}\right]_1^\infty = \frac{1}{1-p}(\infty - 1) = \infty$$

$\therefore \int_1^\infty f(x)\,dx$ diverges at $p<1$ $\Rightarrow \Sigma u_n$ is divergent

(ii) When $p=1$, $f(x) = \frac{1}{x}$

$$\int_1^\infty \frac{1}{x}\,dx = \left[\log x\right]_1^\infty = \infty - 0 = \infty$$

$\int_1^\infty f(x)\,dx$ diverges at $p=1$

$\Rightarrow \Sigma u_n$ is divergent

Hence Σu_n diverges for $p \le 1$

(ii) Examine series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$ for convergence.

Solⁿ: Leaving aside the first term

$$u_n = \frac{n^n}{(n+1)^{n+1}} = \frac{n^n}{n^{n+1}\left(1+\frac{1}{n}\right)^{n+1}} = \frac{1}{n\left(1+\frac{1}{n}\right)^{n+1}}$$

Let $v_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n+1}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} \cdot \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)}$$

$$= \frac{1}{e} \cdot 1$$

Which is finite & $\neq 0$

$\therefore \sum u_n$ & $\sum v_n$ converge or diverge together

$\because \sum v_n = \sum \frac{1}{n}$ is of the form $\sum \frac{1}{n^p}$ with $p = 1$

$\sum v_n$ is divergent

$\Rightarrow \sum u_n$ is divergent

May - 2020

(i) Examine $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots$

Solution: Let $\sum_{n=1}^{\infty} a_n = \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots$

$$= \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

Here $a_n = \frac{1}{n(n+1)(n+2)}$; $a_{n+1} = \frac{1}{(n+1)(n+2)(n+3)}$

$$\frac{a_n}{a_{n+1}} = \frac{1}{n\cancel{(n+1)}\cancel{(n+2)}} \times \cancel{(n+1)}\cancel{(n+2)}(n+3)$$

$$= \frac{n+3}{n} = 1+3/n$$

By Ratio test $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \lim_{n\to\infty}(1+3/n) = 1$

Ratio test fails.

By Raabe's test

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right) = \lim_{n\to\infty} n\left[\cancel{1}+3/n-\cancel{1}\right] = \lim_{n\to\infty} n(3/n) = 3 > 1$$

By Raabe's test; $\sum a_n$ is convergent.

(ii) Examine the series $\frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+\cdots$ $0<x<1$.

Sol: Since $0<x<1 \Rightarrow \lim_{n\to\infty}(x)^n \to 0$

Let $\sum_{n=1}^{\infty} a_n = \frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+\cdots$

$$= \sum_{n=1}^{\infty} \frac{x^n}{1+x^n}$$

Here $a_n = \frac{x^n}{1+x^n}$ and $a_{n+1} = \frac{x^{n+1}}{1+x^{n+1}}$

$$\frac{a_n}{a_{n+1}} = \frac{x^n}{x^{n+1}}\left(\frac{1+x^{n+1}}{1+x^n}\right) = \frac{1}{x}\left(\frac{1+x^{n+1}}{1+x^n}\right)$$

$$\frac{a_n}{a_{n+1}} = \frac{1}{x}\left(\frac{1+x^{n+1}}{1+x^n}\right)$$

By Ratio test $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \frac{1}{x}\left(\frac{1+x^{n+1}}{1+x^n}\right) = \frac{1}{x}\left(\frac{1+0}{1+0}\right) = \frac{1}{x}$

$$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \frac{1}{x}$$

Σa_n is convergent if $\frac{1}{x} > 1 \Rightarrow x < 1$

Σa_n is divergent if $\frac{1}{x} < 1 \Rightarrow x > 1$

Since we are given $0 < x < 1 \Rightarrow \Sigma a_n$ is convergent.

I (a) Determine vectors ~~if~~ $u = (1,2,3)$ and $v = (7,-4,2)$ are linearly dependent?

Sol: Select K_1 and K_2 such that

$$K_1 u + K_2 v = 0$$

$$K_1(1,2,3) + K_2(7,-4,2) = (0,0,0)$$

$\Rightarrow K_1 + 7K_2 = 0, \quad 2K_1 - 4K_2 = 0, \quad 3K_1 + 2K_2 = 0$

$$\Rightarrow \begin{bmatrix} 1 & 7 \\ 2 & -4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to \frac{R_2}{2} \Rightarrow \begin{bmatrix} 1 & 7 \\ 1 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 7 \\ 0 & -9 \\ 0 & -19 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_3 \to \frac{R_3}{-19}$ $\begin{bmatrix} 1 & 7 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 7 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2/9$

$K_1 + 7K_2 = 0$
$K_2 = 0 \Rightarrow K_1 = 0$

As K_1 and K_2 are 0.
u and v are linearly independent

(b) Solve the system of Equations $3x+y+2z=3$, $2x-3y-z=-3$
$x+2y+z=4$

Sol: We have
$$3x+y+2z=3$$
$$2x-3y-z=-3$$
$$x+2y+z=4$$

- Augmented Matrix $[A:B] = \begin{bmatrix} 3 & 1 & 2 & : & 3 \\ 2 & -3 & -1 & : & -3 \\ 1 & 2 & 1 & : & 4 \end{bmatrix}$

$R_1 \leftrightarrow R_3$
$$\begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 2 & -3 & -1 & : & -3 \\ 3 & 1 & 2 & : & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$$\begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -5 & -3 & : & -11 \\ 0 & -5 & -1 & : & -9 \end{bmatrix}$$

$R_3 \to R_3 - R_2$
$$\begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -5 & -3 & : & -11 \\ 0 & 0 & 2 & : & 2 \end{bmatrix}$$

By Gauss Elimination method
$$x + 2y + z = 4 \quad -(1)$$
$$0x - 5y - 3y = -11 \quad -(2)$$
$$2z = 2 \quad -(3)$$

$\Rightarrow z = 1$

put in (2) $\Rightarrow$ $-5y - 3(1) = -11$
$-5y = -11+3$
$-5y = -8 \Rightarrow y = 8/5$

$\boxed{x = -1/5,\ y = 8/5,\ z = 1}$

$x + 2(8/5) + 1 = 4$

$x = 4 - 1 - \frac{16}{5} \Rightarrow 3 - \frac{16}{5} \Rightarrow -1/5$.

8 Find Characteristic Equation of matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and compute A^{-1}. Also express the matrix represented $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.

Sol: Given Matrix is $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

Characteristic Equation $|A - mI| = 0$ where $m =$ eigen value

$$|A - mI| = \begin{vmatrix} 1-m & 4 \\ 2 & 3-m \end{vmatrix} = 0 \Rightarrow (1-m)(3-m) - 8 = 0$$

$$m^2 - 4m + 3 - 8 = 0$$

$$\Rightarrow m^2 - 4m - 5 = 0$$

By Cayley-Hamilton thm: Every square matrix satisfies its ch eq.

$$\Rightarrow A^2 - 4A - 5I = 0 \quad -(1)$$

To find A^{-1}: PreMultiply (1) with A^{-1}

$$A^{-1}(A^2 - 4A - 5I) = 0 \Rightarrow A - 4AA^{-1} - 5A^{-1} = 0$$

$$\Rightarrow 5A^{-1} = A - 4I$$

$$5A^{-1} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 5A^{-1} = \begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$$

Now $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = A^5 - 4A^4 - 5A^3 - 2A^3 + 11A^2 - A - 10I$

$$\Rightarrow A^3(A^2 - 4A - 5I) - 2A^3 + 8A^2 + 3A^2 - A - 10I$$

$$\Rightarrow \cancel{A^3(0) - 2(A} \quad A^3(0) - 2A^3 + 8A^2 + 3A^2 - 10A + 5A - 10I$$

$$\Rightarrow A^3(0) - 2A(A^2 - 4A - 5I) + 3A^2 + 5A - 10I$$

$$= A^3 0 - 2A(0) + 3A^2 + 5A - 10I$$

$$\Rightarrow 3A^2 + 5A - 10I$$

$$= 3A^2 - 12A + 17A - 15I + 5I$$

$$= (3A^2 - 12A - 5I) + 17A + 5I$$

$$= 3(A^2 - 4A - 5I) + 17A + 5I = 17A + 5I$$

$$^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = 17A + 5I$$

$$= 17\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 17+5 & 68 \\ 34 & 51+5 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 68 \\ 34 & 56 \end{bmatrix}$$